Solutions

Math 2D Quiz 4 Morning - February 11th

Please put name and ID on *both* sides for grading and redistribution!

Show all of your work. *There is a question on the back side.

1. Prove whether or not the following limit exists:

$$\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2(y)}{x^3+2y^3}.$$

Hint: Use polar coordinates. $(x = r\cos\theta,\ y = r\sin\theta)$

+2

Must mention at least!

since $\sin(r\sin\theta) \approx r\sin\theta$

$$\lim_{r\to 0} \frac{\cancel{r^2}\cos^2\theta\,\sin^2(r\sin\theta)}{\cancel{r^3}\,(\cos^3\theta+2\sin^3\theta)} = \lim_{r\to 0}\frac{\sin^2(r\sin\theta)\,\cos^2\theta}{r\,(\cos^3\theta+2\sin^3\theta)} \;(=0).$$

☆ Recall: $\sin(x) \approx x$ when x is small! Formally, L'Hopital, differentiate in the $\underline{r}$ variable,

L'Hopital

(chain rule)

$$= \lim_{r\to 0}\frac{2\sin(r\sin\theta)\cos(r\sin\theta)\sin\theta\cos^2\theta}{1\cdot(\cos^3\theta+2\sin^3\theta)} = \frac{2\sin(0)\cos(0)\cos^2\theta\sin\theta}{\cos^3\theta+2\sin^3\theta}$$

$$= \boxed{0}$$ Independent of θ!

+3

Thus it exists (and equals 0)

1 pt each

2. Let $G(x,y) = \frac{y}{x^2+y^2}$. You will be computing all sorts of derivatives.
(a) Compute the first partials: G_x and G_y.
(b) Compute the second partials: G_{xx}, G_{yy}, and G_{xy}.
For this function, $G_{yx} = G_{xy}$ so you may compute G_{yx} instead of G_{xy} if you wish.
(c) **Bonus** Determine if G satisfies $\Delta G = G_{xx} + G_{yy} = 0$.
This time, if you are missing any credit on the quiz at all, part (c) is 1 extra credit point.

a) $G_x(x,y) = \dfrac{0 - y(2x)}{(x^2+y^2)^2} = -\dfrac{2xy}{(x^2+y^2)^2}$.

$G_y(x,y) = \dfrac{1(x^2+y^2) - y(2y)}{(x^2+y^2)^2} = \dfrac{x^2-y^2}{(x^2+y^2)^2}$.

b) $G_{xx}(x,y) = \dfrac{-2y(x^2+y^2)^{\cancel{2}} + (+2xy)\cdot 2\cdot 2x\,\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{4}\,3}} = \dfrac{-2y^3+6x^2y}{(x^2+y^2)^3}$.

$G_{yy}(x,y) = \dfrac{-2y(x^2+y^2)^{\cancel{2}} - (x^2-y^2)\cdot 2\cdot 2y\,\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{4}\,3}} = \dfrac{2y^3-6x^2y}{(x^2+y^2)^3}$.

$G_{xy}(x,y) = \dfrac{-2x(x^2+y^2)^{\cancel{2}} + (+2xy)\cdot 2\cdot 2y\,\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{4}\,3}} = \dfrac{-2x^3+6xy^2}{(x^2+y^2)^3}$

Alternative: $G_{yx}(x,y) = \dfrac{2x(x^2+y^2)^{\cancel{2}} - (x^2-y^2)\cdot 2\cdot 2x\,\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{4}\,3}} = \dfrac{-2x^3+6xy^2}{(x^2+y^2)^3}$.

c) We see $G_{xx} + G_{yy} = \dfrac{-2y^3+6x^2y}{(x^2+y^2)^3} + \dfrac{2y^3-6x^2y}{(x^2+y^2)^3} = 0$ ✓

(or, note $G_{xx} = -G_{yy}$ from (b))